```
> restart;
```

$\mu = cosTheta$

```
> Phg:=(1-g^2)/(1+g^2-2*g*mu)^(3/2);
```

$$Phg := \frac{1-g^2}{(1+g^2-2\,g\,\mu)^{3/2}} \qquad (1)$$

```
> Int(Int(P_HG*mu,mu=-1..1),phi=0..2*Pi)/Int(Int(P_HG,mu=-1..1),phi=0..2*Pi)=int(int(Phg*mu,mu=-1..1),phi=0..2*Pi)/(4*Pi) assuming g<1, g>-1;
```

$$\frac{\int_0^{2\pi}\int_{-1}^{1} P\_HG\,\mu\,d\mu\,d\phi}{\int_0^{2\pi}\int_{-1}^{1} P\_HG\,d\mu\,d\phi} = g \qquad (2)$$

Hence <cosTheta> = g

```
> int(int(Phg,mu=-1..1),phi=0..2*Pi)/(4*Pi) assuming g<1, g>-1;
```

$$1 \qquad (3)$$

With the CornettShanks modified HG phase function

```
> Phg_CS:=3/2*(1+mu^2)*Phg/(2+g^2);
```

$$Phg\_CS := \frac{3}{2}\,\frac{(1+\mu^2)\,(1-g^2)}{(1+g^2-2\,g\,\mu)^{3/2}\,(2+g^2)} \qquad (4)$$

```
> Int(Int(P_HG_CS*mu,mu=-1..1),phi=0..2*Pi)/Int(Int(P_HG_CS,mu=-1..1),phi=0..2*Pi)=(int(int(Phg_CS*mu,mu=-1..1),phi=0..2*Pi) assuming g<1, g>-1)/(int(int(Phg_CS,mu=-1..1),phi=0..2*Pi) assuming g<1, g>-1);
```

$$\frac{\int_0^{2\pi}\int_{-1}^{1} P\_HG\_CS\,\mu\,d\mu\,d\phi}{\int_0^{2\pi}\int_{-1}^{1} P\_HG\_CS\,d\mu\,d\phi} = \frac{3}{5}\,\frac{g\,(g^2+4)}{2+g^2} \qquad (5)$$

Hence now <cosTheta>= $\frac{3\,g\,(g^2+4)}{5\,(g^2+2)}$

For g -> 1 <cosTheta> -> g , i.e. Phg_CS -> Phg

```
> X:=5/9*mu + 125/729*mu^3+sqrt(64/27-325/243*mu^2+1250/2187*mu^4);
```

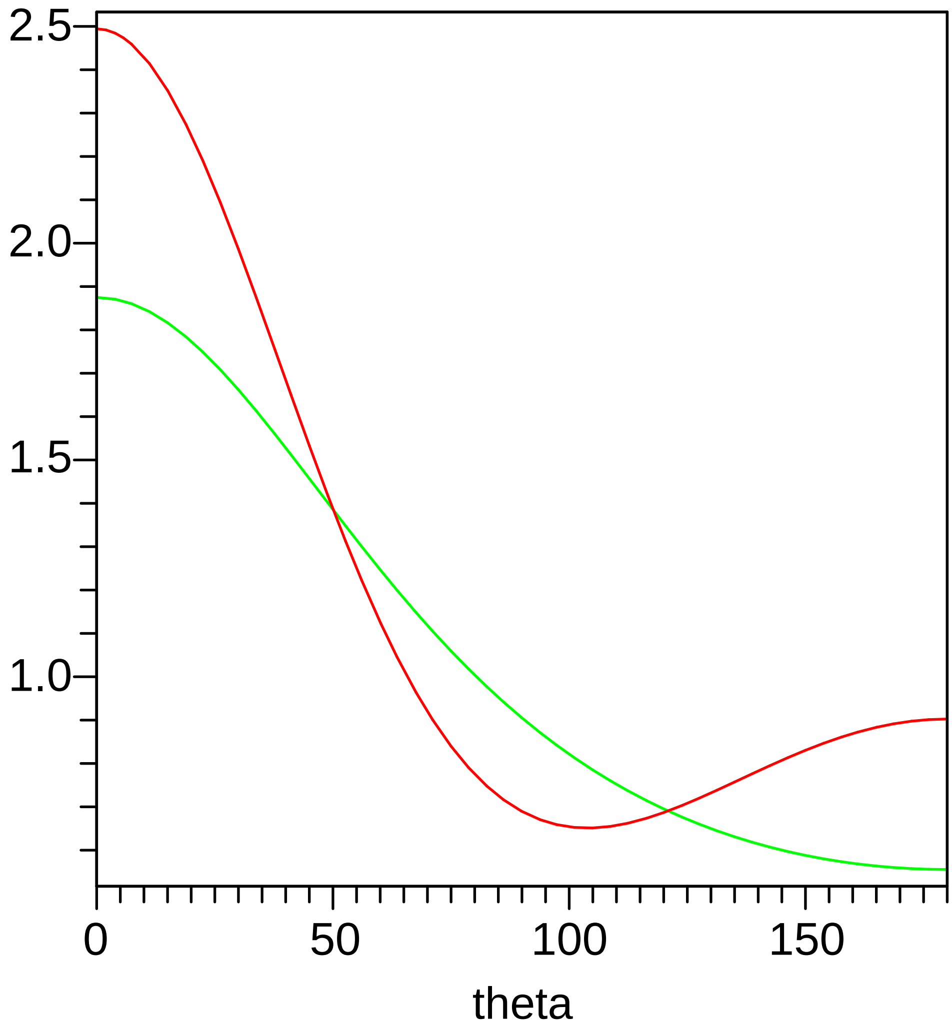
(6)

$$X := \frac{5}{9}\mu + \frac{125}{729}\mu^3 + \frac{1}{81}\sqrt{15552 - 8775\mu^2 + 3750\mu^4} \qquad (6)$$

```
> G:=subs(x=X,5/9*mu -(4/3-25/81*mu^2)*x^(-1/3) + x^(1/3));
```

$$G := \frac{5}{9}\mu - \frac{\frac{4}{3} - \frac{25}{81}\mu^2}{\left(\frac{5}{9}\mu + \frac{125}{729}\mu^3 + \frac{1}{81}\sqrt{15552 - 8775\mu^2 + 3750\mu^4}\right)^{1/3}} + \left(\frac{5}{9}\mu + \frac{125}{729}\mu^3 + \frac{1}{81}\sqrt{15552 - 8775\mu^2 + 3750\mu^4}\right)^{1/3} \qquad (7)$$

```
> with(plots):
> p1:=plot(subs(mu=cos(theta*Pi/180),g=0.2,Phg),theta=0..180,color=
  green):
> p2:=plot(subs(mu=cos(theta*Pi/180),g=G,mu=0.2,Phg_CS),theta=0.
  .180,color=red):
> display({p1,p2},axes=boxed);
```



Backscattering fraction

>